

2.5

What's the Point?

Deriving Quadratic Functions

LEARNING GOALS

In this lesson, you will:

- Determine how many points are necessary to create a unique quadratic equation.
- Derive a quadratic equation given a variety of information using reference points.
- Derive a quadratic equation given three points using a system of equations.
- Derive a quadratic equation given three points using a graphing calculator to perform a quadratic regression.

No matter where you go to see professional baseball in the United States, the dimensions of the infield are all the same. The bases will always be ninety feet apart, and the pitcher's mound is 60.6 feet from home plate. The outfield, however, is a much different story! In fact, baseball is unique in that outfield dimensions, foul area, and outfield walls can be different depending on where the game is being played. For example, the distance from home plate to the right field wall in old Yankee Stadium was actually closer to home plate than its left field wall, making their left-handed power hitters very happy.

Have you ever seen an outfielder jump high above the left field wall to catch a potential homerun? They won't be able to do this in Boston, where the left field wall is 37 feet tall—earning the nickname “the Green Monster.”

The dimensions of the field aren't the only differences you will find from stadium to stadium. Dodger Stadium in Los Angeles is built into a mountain side called Chavez Ravine. Turner Field in Atlanta has an arcade area called Scouts Alley. Coors Field in Denver is one mile above sea level that allows the fly balls to travel much further than they normally would because of the thin, dry air. The outfield of PNC Park in Pittsburgh opens up along the Monongahela River, making the city's beautiful skyline visible to all spectators.

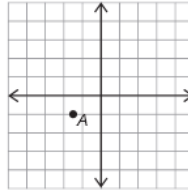
Have you ever been to a Major League Baseball game? Did you notice anything unique about the stadium?

PROBLEM 1 A Dime A Dozen

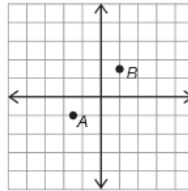


1. Consider the family of linear functions. Use the given point(s) to sketch possible solutions.

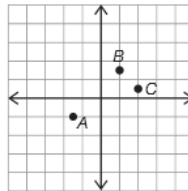
a. How many lines can you draw through point A?



b. How many lines can you draw through both points A and B?



c. How many lines can you draw through all points A, B, and C?



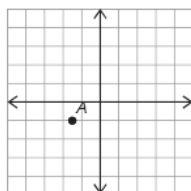
2. What is the minimum number of points you need to draw a unique line?

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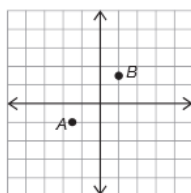
3. Consider the family of quadratic functions. Use the given point(s) to sketch possible solutions.

a. How many parabolas can you draw through point A ?

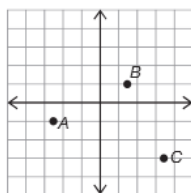


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b. How many parabolas can you draw through both points A and B ?



c. How many parabolas can you draw through all points A , B , and C ?





4. Use each coordinate plane and the given information to draw possible parabolas for Examples A through J.

If there is more than one parabola, draw it.

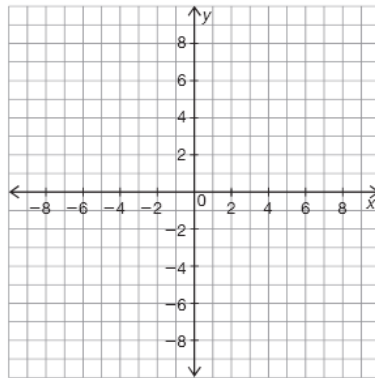


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Example A

Given information: The vertex is $(-3, 4)$.

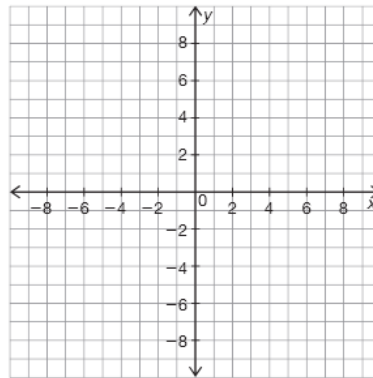
Sketch:



Example B

Given information: The vertex is $(-3, 4)$ and $(-4, 1)$ is a point on the parabola.

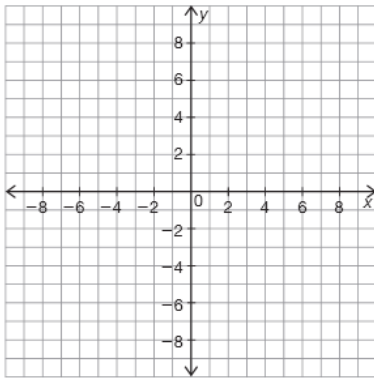
Sketch:



Example C

Given information: The vertex is $(3, -2)$ and one of the two x -intercepts is $(4, 0)$.

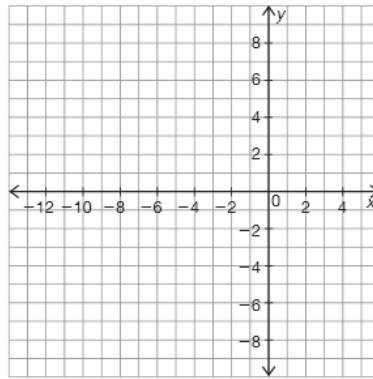
Sketch:



Example D

Given information: The parabola has exactly one x -intercept at $(-4, 0)$ and a y -intercept at $(0, 4)$.

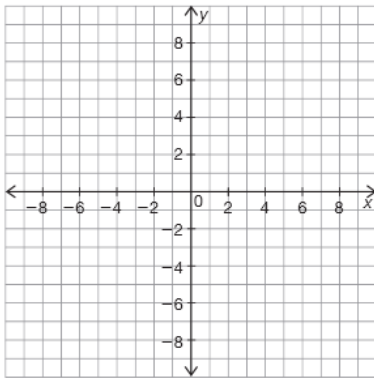
Sketch:



Example E

Given information: The x -intercepts are $(-2, 0)$ and $(2, 0)$.

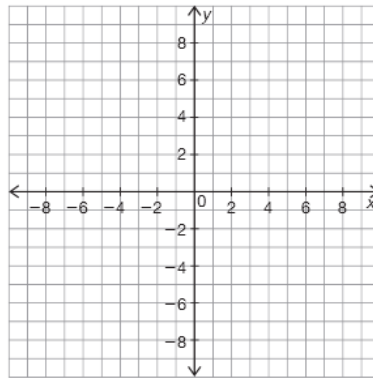
Sketch:



Example F

Given information: The x -intercepts are $(-2, 0)$ and $(2, 0)$, and $(-1, -6)$ is a point on the parabola.

Sketch:

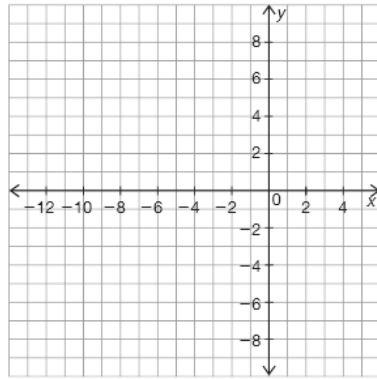


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Example G

Given information: The axis of symmetry is $x = -5$ and $(-3, 6)$ is a point on the parabola.

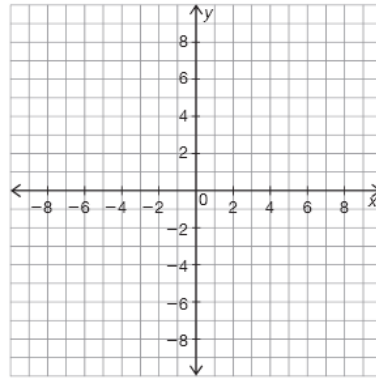
Sketch:



Example H

Given information: The axis of symmetry is $x = -5$, and $(-3, 6)$ and $(1, -10)$ are two points on the parabola.

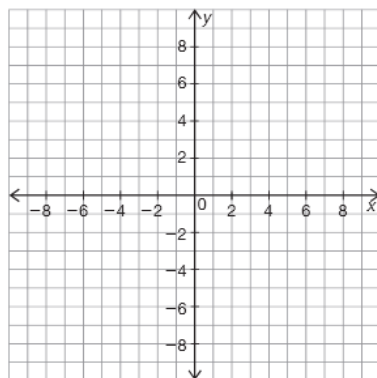
Sketch:



Example I

Given information: Three points on the parabola are $(2, 2)$, $(3, 4)$, and $(4, 6)$.

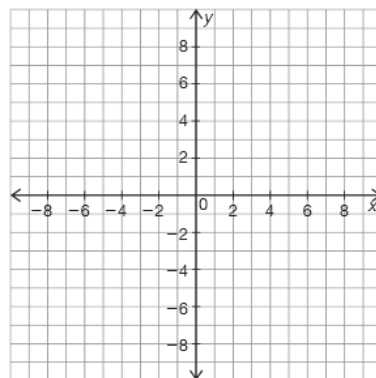
Sketch:



Example J

Given information: Three points on the parabola are $(-4, -8)$, $(0, 8)$, and $(7, -2.5)$.

Sketch:



5. Review your work in Question 4.
 - a. List the example(s) in which you could draw more than one parabola. How many points were you given for the example(s) you listed?

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- b. List the example(s) in which you could draw only one parabola. How many points were you given for the example(s) you listed?



- c. List the example(s) in which you could not draw a parabola. What did you notice that was different from the examples in which you were able to draw one or more parabolas?



- 6. Consider the examples in which you could draw only one parabola.
 - a. Enter the example letter (A through J) and the given information in the appropriate columns of the table shown.
 - b. For each example you listed, did you use the given information to determine any other points so that you could draw the parabola? If yes, enter the number of additional points you used in the appropriate column.
 - c. Enter the total number of points you used to draw each parabola in the appropriate column.

| Information that Determines A Unique Parabola | | | | |
|---|-------------------|------------------------|-----------------------------|------------------------|
| Example | Given Information | Number of Points Given | Number of Additional Points | Total Number of Points |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |



- 7. Summarize your results from Questions 1 through 6 to write a rule for the total number of points needed to determine a unique parabola.

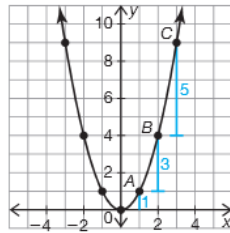
PROBLEM 2 One of a Kind



Now that you learned how to determine a unique graph of a quadratic function, let's explore ways to write the function.

To write the quadratic function you will need to use the reference points and consider the vertical distance between each point. The basic quadratic function, the reference points, and the vertical distance between those points is shown.

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You can use reference points and the factored form of a quadratic function to write the function for the graph explained in Example F. The x -intercepts are $(-2, 0)$ and $(2, 0)$, and $(-1, -6)$ is a point on the parabola.

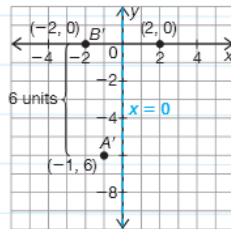
First, determine the axis of symmetry.

The axis of symmetry must be directly in the middle of the two x -intercepts.

$$x = \frac{r_1 + r_2}{2} = \frac{-2 + 2}{2} = \frac{0}{2} = 0$$

The axis of symmetry is $x = 0$.

Plot the points and label the axis of symmetry on a coordinate plane.



Point B' is two units away from the axis of symmetry, therefore corresponds to reference point B on the basic function.

Point A' is one unit away from the axis of symmetry, therefore corresponds to reference point A on the basic function.

Next, determine the a -value.

In the basic quadratic function, the vertical distance between reference point A and B is 3 units. The distance between A' and B' is 6 or 3×2 units, therefore $a = 2$.

You now know the values of r_1 , r_2 , and a , so you can create the unique quadratic function.

$$f(x) = a(x - r_1)(x - r_2)$$

$$f(x) = 2(x - 2)(x + 2)$$

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1. Brayden, Max, and Ian are each writing quadratic functions that must satisfy the given information: x-intercepts at (3,0) and (9,0).

Brayden

I know the axis of symmetry is $x = 6$ because it is in the middle of the two x-intercepts. I randomly choose a y-value of 9 to complete the vertex-form.

$$\begin{aligned} f(x) &= a(x - h)^2 + k \\ 0 &= a(3 - 6)^2 + 9 \\ 0 &= a(-3)^2 + 9 \\ 0 &= 9a + 9 \\ -9 &= 9a \\ a &= -1 \\ \text{Therefore } f(x) &= -1(x - 6)^2 + 9 \end{aligned}$$

Max

I know the values for r_1 and r_2 . So, all I need is the a-value. I randomly choose an a-value of 4. The function is:

$$f(x) = 4(x - 3)(x - 9)$$

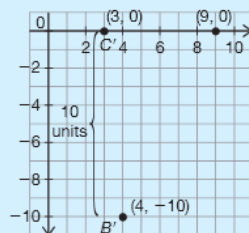
Ian

I created a graph to model the situation, and choose to add the point (4, -10).

I know that the given point (3, 0) must be the C' because it is 3 units away from the axis of symmetry. By the same reasoning my new point must be B' because it is only 2 units away from the axis of symmetry. If the a-value was 1, the vertical distance between B' and C' would be 5. In this graph the vertical distance is 2×5 , therefore the a-value must be 2.

The function is:

$$f(x) = 2(x - 3)(x - 9)$$



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Who's method and quadratic function is correct? Explain your reasoning.

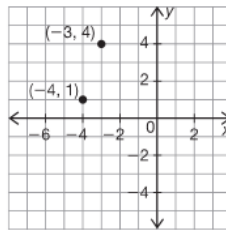


2. Use your knowledge of reference points to write a quadratic function for the examples previously identified as unique parabolas in Problem 1. If it is not possible to write a function, state why not.

a. Example B

Given: Vertex $(-3, 4)$; point $(-4, 1)$

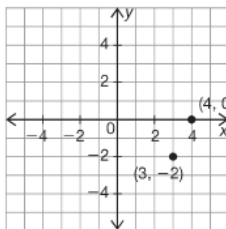
$f(x) =$ _____



b. Example C

Given: Vertex $(3, -2)$; one of two x -intercepts $(4, 0)$

$f(x) =$ _____

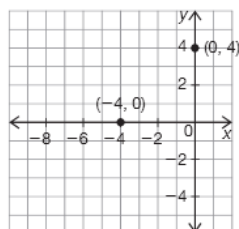


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c. Example D

Given: Exactly one intercept $(-4, 0)$; y -intercept $(0, 4)$

$f(x) = \underline{\hspace{2cm}}$



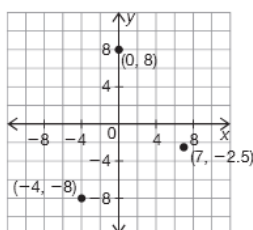
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d. Example J

Given: Points $(-4, -8)$, $(0, 8)$, $(7, -2.5)$

$f(x) = \underline{\hspace{2cm}}$



PROBLEM 3 Technology Whiz

Using reference points and the axis of symmetry to write the equation of a quadratic function is an efficient method to use when given certain points. However, this method will not always work. As you saw in Example J of Problem 2, if you do not know the axis of symmetry, you cannot use reference points. You can use a graphing calculator or systems of equations as two other methods to write quadratic functions.

Make sure you set the appropriate viewing window to view the graph.

**2**

You can use a graphing calculator to determine a quadratic regression equation given three points on the parabola.

Step 1: Diagnostics must be turned on so that all needed data is displayed. Press **2nd CATALOG** to display the catalog. Scroll to **DiagnosticOn** and press **ENTER**. Then press **ENTER** again. The calculator should display the word **Done**.

Step 2: Press **STAT** and then press **ENTER** to select **1:Edit**. In the **L1** column, enter the x-values by typing each value followed by **ENTER**. Use the right arrow key to move to the **L2** column. **ENTER** the y-values.

Step 3: Press **STAT** and use the right arrow key to show the **CALC** menu. Choose **5:QuadReg**. Press **Enter**. The values for a, b, and c will be displayed.

Step 4: To have the calculator graph the exact equation, press **Y=**, **VARs**, **5:Statistics**, scroll right to **EQ**, press **1:RegEQ**, **GRAPH**.

If there is already data in your L1 list, highlight the heading L1, Press **CLEAR**, then Press **ENTER** to delete it.

1. Use a graphing calculator to determine the quadratic equation for Example J.



2. Use a graphing calculator to determine the quadratic function for each set of three points that lie on a parabola.

a. points $(-1, 36)$, $(1, 12)$, and $(2, 6)$

b. points $(0, 2)$, $(-1, 9)$ and $(3, 5)$

c. points $(2, 3)$, $(3, 13)$ and $(4, 29)$



3. Van McSluggler needs one more homerun to advance to the next round of the home run derby. On the last pitch, he takes a swing and makes contact. Initially, he hits the ball at 5 feet in the air. At 32 feet from home plate his ball was 23.7 feet in the air, and at 220 feet from home plate his ball was 70 feet in the air.

a. Draw a figure to represent this situation. Include any known data points.

b. Use a graphing calculator to write a function for the height of the ball in terms of its horizontal distance to home plate. Round to the nearest thousandth.

c. If Van's ball needs to travel a distance of 399 feet in order to get the homerun, did he succeed? Explain why or why not.



d. What was the maximum height of Van's baseball?

PROBLEM 4 All About the Algebra

You know that the method of using reference points to determine a quadratic equation does not always work. You know how to use a graphing calculator to create the equation, but what happened before the graphing calculator? What if you don't have graphing calculator, or needed to explain to somebody how to write the equation?

You can use algebra to solve! You can set up and solve systems of equations to determine a quadratic equation.

2

You now know that you need a minimum of 3 non-linear points to create a unique parabola. In order to create an equation to represent the parabola, you must use systems of equations.

Consider the three points $A(2, 1)$, $B(-1, -2)$, and $C(3, -10)$.

First, create a quadratic equation in the standard form $y = ax^2 + bx + c$ for each of the points:

| | |
|---|---------------------------------|
| Point A: $1 = a2^2 + b2 + c$ $1 = 4a + 2b + c$ | Equation A: $1 = 4a + 2b + c$ |
| Point B: $-2 = a(-1)^2 + b(-1) + c$ $-2 = a - b + c$ | Equation B: $-2 = a - b + c$ |
| Point C: $-10 = a3^2 + b3 + c$ $-10 = 9a + 3b + c$ | Equation C: $-10 = 9a + 3b + c$ |

Now, use elimination and substitution to solve for a , b , and c .

STEP 1: Subtract Equation B from A:

$$\begin{array}{r} 1 = 4a + 2b + c \\ -(-2 = a - b + c) \\ \hline 3 = 3a + 3b \end{array}$$

STEP 2: Subtract Equation C from B:

$$\begin{array}{r} -10 = 9a + 3b + c \\ -(-2 = a - b + c) \\ \hline -8 = 8a + 4b \end{array}$$

STEP 3: Solve the equation from Step 1 in terms of a .


$$\begin{array}{r} 3 = 3a + 3b \\ 3a = 3 - 3b \\ a = 1 - b \end{array}$$

STEP 4: Substitute the value for a into the equation from Step 2.




$$\begin{array}{r} -8 = 8(1 - b) + 4b \\ -8 = 8 - 4b \\ 4b = 16 \\ b = 4 \end{array}$$

STEP 5: Substitute the value for b into the equation from Step 3.



$$\begin{array}{r} a = 1 - (4) \\ a = -3 \end{array}$$



STEP 6: Substitute the values for a and b into Equation A.

$$1 = 4a + 2b + c$$
$$1 = 4(-3) + 2(4) + c$$
$$1 = -12 + 8 + c$$
$$1 = -4 + c$$
$$5 = c$$


STEP 7: Substitute the values for a , b , and c into the standard form of a quadratic.

$$y = -3x^2 + 4x + 5$$


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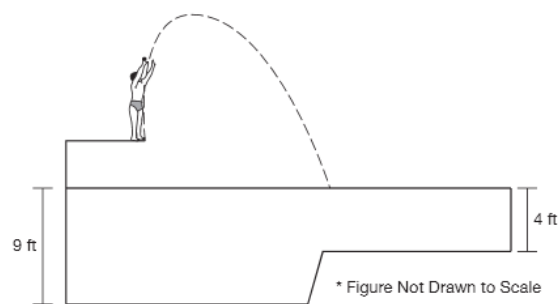
1. Create a system of equations and use algebra to create a quadratic equation with points $(-1, 5)$, $(0, 3)$, and $(3, 9)$.

2

2. Happy Homes Development Company has hired Splish Splash Pools to create the community pool for their new development of homes. The rectangular pool is to have one section with a 4-foot depth, and another section of a depth of 9 feet. The pool will also have a diving board. By law, the regulation depth of water necessary to have a diving board is 9 feet. Happy Homes would like to have the majority of the pool to be a 4-foot depth in order to accommodate a large number of young children.

The diving board will be 3 feet above the edge of the pool's surface and extend 5 feet into the pool. After doing some research, Splish Splash Pools determined that the average diver would be 10 feet in the air when he is 6 feet from the edge of the pool, and 11 feet in the air when he is 8 feet from the edge of the pool. According to this dive model, what is the minimum length of 9 foot depth section of the pool?

- a. Fill in the diagram with all known information.



- b. Write a quadratic equation in standard form for each of the points you know.

- c. Use substitution and elimination to solve the system of equations for variables a , b , and c .



- d. Use your new equation to determine the minimum length of the 9 foot depth section of the pool.



Be prepared to share your solutions and methods.